SPECT3D Benchmark Calculation:
Radiative Transfer in 1-D

This memo describes benchmark calculations for 1-D multi-angle radiative transfer models used in SPECT3D. A multi-angle radiative transfer model for 1-D cylindrical geometries has recently been added to SPECT3D. This complements existing multi-angle models for 1-D planar and 1-D spherical geometries. Below, we compare SPECT3D results with exact solutions for all three geometries.

In the calculations below, we assume the plasma to be spatially uniform. All calculations are done at a single frequency point. The size of the plasma is specified by the optical depth, τ_0. For planar geometry, τ_0 corresponds to the slab width (see Figure 1). For cylindrical and spherical geometries, τ_0 corresponds to the radius.

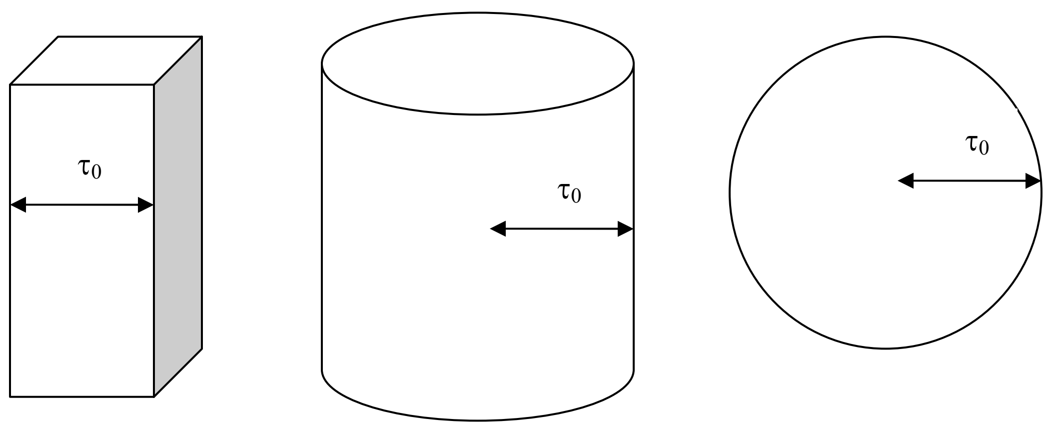

Figure 1. Schematic illustration of plasma geometries with size characterized by optical depth τ_0.

The photoionization and photoexcitation rates at a given location within a plasma depend on the mean intensity at that point. The mean intensity, J_ν, at a point *r* in a spatial grid is given by the angle average of the specific intensity I_ν:

$$J_\nu(r) = \frac{1}{4\pi}\int_{-1}^{1} d\mu \int_{0}^{2\pi} d\varphi \; I_\nu(r,\mu,\varphi) \qquad (1)$$

where μ is the cosine of the polar angle, and φ is that azimuthal angle.

In cylindrical geometry, with a spatially uniform source function, S_ν, the exact solution is:

$$\frac{J_\nu}{S_\nu} = \frac{1}{4\pi}\int_{0}^{2\pi} d\varphi \int_{0}^{1} d\mu \left[2 - e^{-T_A(\mu,\varphi)} - e^{-T_B(\mu,\varphi)}\right] \qquad (2)$$

where:

$$T_A(\mu,\varphi) = \frac{\tau_0}{\sqrt{1-\mu^2}} \left\{ \sqrt{1-\gamma^2 \sin^2(\varphi)} \; + \; \gamma \cos(\varphi) \right\} \tag{3}$$

and

$$T_B(\mu,\varphi) = \frac{\tau_0}{\sqrt{1-\mu^2}} \left\{ \sqrt{1-\gamma^2 \sin^2(\varphi)} \; - \; \gamma \cos(\varphi) \right\} \tag{4}$$

where γ is the scaled radius, $r/R = \tau/\tau_0$.

Comparisons between SPECT3D results and exact solutions for 1-D cylindrical geometry are shown in Figure 2. In the SPECT3D calculations, we used a multi-angle grid with 2 μ–points and 10 φ–points (*i.e.,* a total of 20 angles). Results are shown for several optical depth cases, ranging from optically thin (τ_0 = 0.000184) to optically thick (τ_0 = 170) plasmas. In each case, the agreement between the SPECT3D results and the exact solution is good, with typical differences being ~ a few percent.

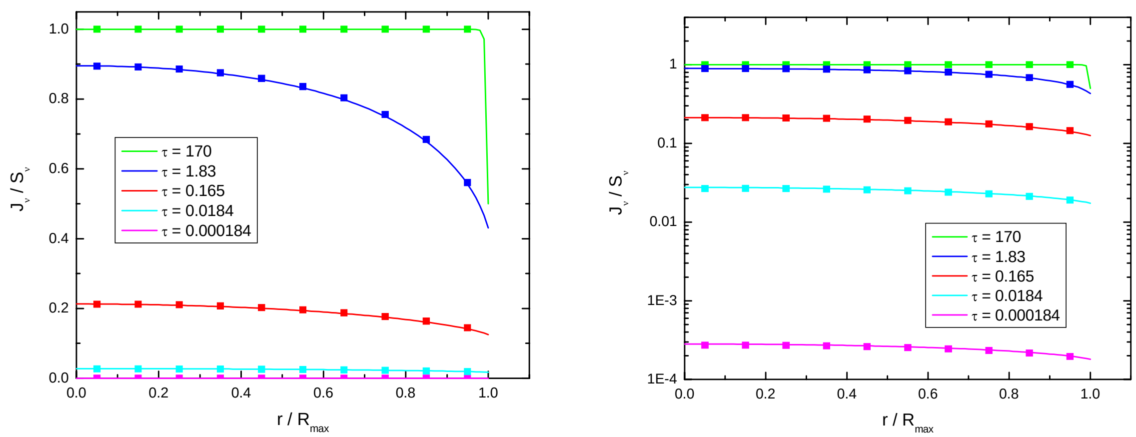

Figure 2. Calculated mean intensity as a function of position for uniform 1-D cylindrical plasmas. The radius of the plasma is characterized by the optical depth τ_0. The SPECT3D results (symbols) are compared with exact solutions (solid curves). Left: J_ν / S_ν on a linear scale. Right: Same results on a log scale.

In planar geometry, the exact solution is [1]:

$$\frac{J_\nu}{S_\nu} = 1 - \frac{1}{2} E_2(\tau) - \frac{1}{2} E_2(\tau_0 - \tau) \tag{5}$$

while in spherical geometry, the exact solution is:

$$\frac{J_\nu}{S_\nu} = \frac{1}{2} \int_0^1 d\mu \left\{ 2 - e^{T_B(\mu)} - e^{T_A(\mu)} \right\} \tag{6}$$

where $T_A(\mu)$ and $T_B(\mu)$ are given by:

$$T_A(\mu) = \tau_0 \left\{ \sqrt{1-\gamma^2\mu^2} \; + \; \gamma \, \mu \right\} \tag{3}$$

and

$$T_B(\mu) = \tau_0 \left\{ \sqrt{1-\gamma^2\mu^2} \; - \; \gamma \, \mu \right\}. \tag{3}$$

Comparisons between SPECT3D results and exact solutions for planar, cylindrical, and spherical geometries are shown in Figure 3. For each calculation, the optical depth was τ_0 = 0.165. In the SPECT3D calculations, we used a multi-angle grid with 5 angles in planar geometry, 10 angles in spherical geometry, and 20 angles in cylindrical geometry. Again, we see the SPECT3D results agree with the exact solutions to within ~ a few percent. Note that the mean intensity in the cylindrical case is a factor of 2 to 3 higher even though the radius of the sphere and cylinder are the same. This is because the cylinder extends to infinity along the *z*-axis (*i.e.*, the cylinder has more volume and the sphere would fit inside the cylinder.)

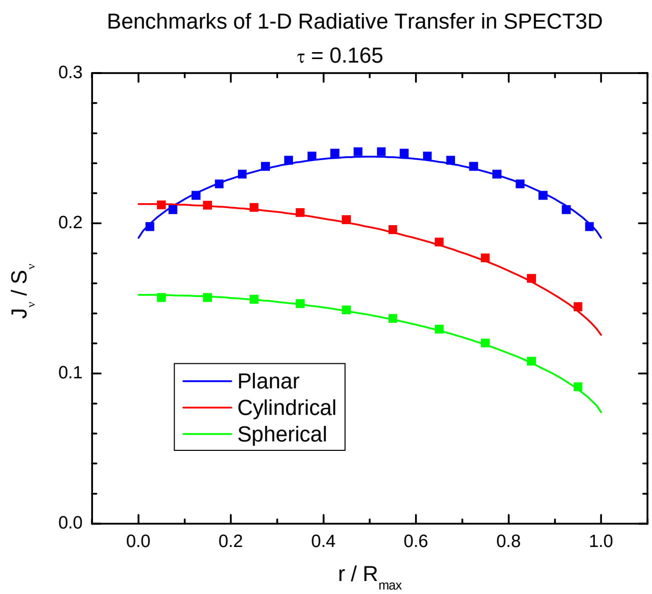

Figure 3. Spatial variation in mean intensity for planar, cylindrical, and spherical plasmas calculated using multi-angle radiative transfer models in SPECT3D. In each case, the radius of the plasma is characterized by an optical depth of τ_0 = 0.165. The SPECT3D results (symbols) are compared with exact solutions (solid curves).

Future work will include extending the multi-angle radiative transfer modeling in SPECT3D to 2-D cylindrical R-Z geometry.

References

[1] Mihalas, D., Stellar *Atmospheres, Second Edition*, Freeman and Co., New York (1978).